



Application of Model Forecasting to Generate Real Gross Domestic Product Data

A.A. Agboluaje

Department of Mathematical Sciences, Faculty of Natural Sciences, IBB University, Lapai., Nigeria

ABSTRACT

The objective is to provide the right model forecast for the estimation of the Real Gross domestic product (RGDP) data among the Autoregressive Conditional Heteroscedastic [ARCH] series models for better decision making. ARCH series of models were applied using maximum likelihood estimators assuming Gaussian distribution, Student t- distribution and Generalized Error Distribution (GED). Exponential Generalized Autoregressive Conditional Heteroskedasticity-Generalized Error Distribution (EGARCH-GED) has good distribution fit and the best model fit. The dynamic and static forecast evaluation results revealed that Exponential Generalized Autoregressive Conditional Heteroskedasticity-Student t- Distribution (EGARCH-ST) has minimum forecast error which was the best forecast accuracy which indicated the best model forecast. The inequality coefficients showed low fitness, there were little biasness proportions, large variance proportions and low values of covariance proportions which presented fairly good forecast. Thus, EGARCH-ST is the best model forecast for forecast accuracy using Nigeria RGDP quarterly data for better decision

Keywords: ARCH series of models, best model fit, best model forecast, EGARCH-ST

INTRODUCTION

The objective is to provide the right model forecast for the estimation of the Real Gross domestic product (RGDP) data from among the ARCH series models [Autoregressive Conditional Heteroscedastic (ARCH), Generalised ARCH (GARCH), Threshold GARCH (TGARCH), exponential GARCH (EGARCH), power ARCH (PARCH) and Component GARCH (CGARCH)]. Precise forecast needs the right model forecast for good results and better planning (Engle, 1982). RGDP is the collective economic indicator and is used as a measure for standard of living. RGDP changes when it captured the changes in the economic well-being. RGDP as a measure available in the beginning of 2000 which were worryingly slow in spite of a noticeable developments in high technological equipment like smart phone, internet facilities like Facebook and Google, business models, and health care value (Feldstein, 2017; Dynan & Sheiner, 2018). Modelling the quarterly Nigeria RGDP data is heteroscedastic in nature which requires the right model forecast for better prediction of the standard of living of a society. The estimation of volatility prediction using variance models which are mostly revealed in

financial time series. In order to know the use of the conditional variance models, the conditional and unconditional variances differences are first identified. Therefore, the conditional volatility with its symmetric and asymmetric effects are captured when ARCH series of models are applied (Thapa & Gautam, 2016).

Engle (1982) introduced ARCH model to time varying volatility. ARCH models are capable of holding any variations presented by economic forecaster. ARCH model can hold limited lags. Limited lag and volatility persistence were the weaknesses of ARCH model which was uplifted by GARCH model (Bollerslev, 1986). There were surplus kurtosis and volatility persistence which were the challenges of GARCH model. When the positive and negative shocks of returns with the same magnitude have a right skewed effect on forthcoming volatility, then, the increase in the negative returns reveal the leverage effect (Vivian & Wohar, 2012; Ewing

Received 30 May, 2020

Accepted 30 June, 2020

Address Correspondence to:

ayo_dele1960@yahoo.com,

& Malik, 2013; Martinet & McAleer, 2016; Jiang & Xia, 2018; Chang & McAleer, 2019). The asymmetric effects of positive and negative shocks of the same dimension on conditional volatility in various ways are captured by TGARCH and EGARCH captured the asymmetric effects of positive and negative shocks of the equal dimensions on conditional volatility in several directions. Leverage is a special asymmetry (Harvey, 1993). The EGARCH non-linear model of conditional variance withstand the asymmetric volatility behaviour (Harvey, 1993). Mutunga *et al.* (2015) argued that EGARCH has the minimum mean square error and mean absolute error which has better forecast than TGARCH model.

The correct interpretation of the EGARCH asymmetry is achieved using the regularity condition which concentrates on the correct parameter, α rather than the incorrect parameter, γ (Chang & McAleer, 2017). The stock market and foreign exchange market in Hungary, Poland, the Czech Republic, Romania and Croatia were modelled to reveal the asymmetric volatility spill over effects using EGARCH model (Ngo Thai, 2019). TGARCH, EGARCH, PARCH and CGARCH models are restricted to have the conditional variances positivity, stationarity, and existence of fourth-order moments. EGARCH estimates revealed that the finite kurtosis condition has higher percentage of the series, but PARCH estimates has a low percentage of the series satisfy by the finite kurtosis restriction (Nelson, 1991; Hentschel, 1995; Rodríguez & Ruiz, 2012; McAleer, 2014). Therefore, ARCH series models are employed for better understanding of the right model forecast that would give reliable results.

METHODOLOGY

The source of Real Gross Domestic Product (RGDP) data is National Bureau of Statistics (NBS) and retrieved from the website. RGDP quarterly data from 2010Q₁ to 2019Q₂ is employed to cater for the right model forecast to produce precise forecast. The ARCH model series employed in this study are as follows.

The ARCH model

Engle (1982) offered Autoregressive Conditionally Heteroscedastic (ARCH) model which allowed the variance conditional on the previous. The unconditional variance is constant, while the conditional variance is a

linear function of the squared previous values of the series.

$$x_t = \phi x_{t-1} + \varepsilon_t \tag{1}$$

$$\varepsilon_t \sim N(0, h_t)$$

$$h_t = \omega + \beta_1 \varepsilon_{t-1}^2 + \dots + \beta_q \beta_{t-q}^2 \tag{2}$$

where,

$$q > 0, \omega > 0$$

$$\beta_i \geq 0, i = 1, \dots, q$$

If $q = 0$ then ε_t is white noise.

The ARCH process is in order q and β_i is the unknown parameters.

The GARCH model

Bollerslev (1986) offered Generalized Autoregressive Conditionally Heteroscedastic (GARCH) models which allowed the variance conditional on the previous. The conditional variance is a linear function of the squared previous values of the series in the traditional GARCH models.

$$x_t = \phi x_{t-1} + e_t \tag{3}$$

$$e_t \sim N(0, h_t)$$

$$h_t = \omega + \beta_1 e_{t-1}^2 + \dots + \beta_q e_{t-q}^2 + \gamma_1 h_{t-1} + \dots + \gamma_p h_{t-p} \tag{4}$$

where,

$$p \geq 0, q > 0, \omega > 0$$

$$\beta_i \geq 0, i = 1, \dots, q$$

$$\gamma_i \geq 0, i = 1, \dots, p$$

If $p = 0$, then ARCH (q) process is the procedure and e_t is white noise, if $p = q = 0$.

The TGARCH model

Zakoian (1994) offered TGARCH model with the following specification:

$$\sigma_t^2 = \omega + \gamma |x_{t-1}| + \delta \sigma_{t-1} + \varphi_T x_{t-1} \tag{5}$$

If $\omega > 0, \gamma \geq 0, \delta \geq 0$ and $|\varphi_T| < \gamma$, then the conditional square root of variance is positive.

The EGARCH model

Nelson (1991) offered Exponential GARCH model which have been well-known in modelling the heteroscedastic error and the model is presented as follows:

$$\begin{aligned} \log h_t &= \omega + \partial |x_{t-1}| + \gamma x_{t-1} \\ &+ \varphi \log h_{t-1}, \quad |\varphi| < 1 \end{aligned} \tag{6}$$

where $x_t = e_t / \sqrt{h_t}$ is the standardized shocks and $x_t \sim iid(0, B)$. If $|\varphi| < 1$ then there is stability. The impact is asymmetric if $\gamma \neq 0$ and the leverage exist if $\gamma < 0$ and $\partial < -\gamma$ (Nelson, 1991; McAleer, 2014; McAleer & Hafner, 2014; Martinet & McAleer, 2016).

The PARCH model

Ding *et al.* (1993) introduced the PARCH model as an ARCH series of model (GARCH) of Bollerslev (1986) that has about seven GARCH specifications. The model is specified as

$$\sigma_t = [\mu + \theta_1(|\epsilon_{t-1}| - \theta_n \epsilon_{t-1})^\partial + \phi_1 \sigma_{t-1}^\partial]^{1/\partial} \quad (7)$$

where $\mu, \theta_1, \theta_n, \phi_1$ and ∂ are the parameters to be estimated. $\partial > 0$, act as Box-Cox transformation of σ_t . $\theta_n, (-1 < \theta_n < 1)$ and called leverage effect.

The CGARCH model

Suppose a martingale difference order is μ_t . Then the model is specified by the equations

$$\mu_t = h_t y_t$$

$$h_t^2 = \varphi_t h_{1t}^2 + (1 - \varphi_t) h_{2t}^2 \quad (8)$$

where y_t , for $t = 1, \dots, T$, is an *iid*(0, 1) order of arbitrary variable and

$$h_{1t}^2 = \alpha_{01} + \alpha_{11} \mu_{t-1}^2 + \beta_{11} h_{1,t-1}^2$$

$$h_{2t}^2 = \alpha_{02} + \alpha_{11} \mu_{t-1}^2 + \beta_{22} h_{2,t-1}^2$$

through $(\alpha_{0i}, \alpha_{1i}, \beta_{1i})$ as constant coefficients sustaining the constraints $\alpha_{0i} > 0, \alpha_{1i} > 0$ and $\beta_{1i} > 0, i = 1, 2$. Moreover, suppose that $\varphi_t = \varphi(I_t - 1)$, with $0 \leq \varphi_t \leq 1$, where $I_t - 1$ symbolizes the set of information available at time $(t - 1)$.

The estimation of these ARCH series of models (symmetric and asymmetric) are using

Maximum Likelihood estimators assuming Gaussian distribution, Student *t* Distribution and Generalized Error Distribution (GED).

RESULTS AND DISCUSSION

The behaviour of the graph of the quarterly Real Gross Domestic Product (RGDP) data was trending and not stationary. The observation of kurtosis and skewness revealed the existence of heteroscedasticity in the level data. Jarque-Bera of residual normality tests was significant which indicated that the data were not distributed normally. The confirmation of the heteroscedastic nature of the data were shown in Table 1.

The level data that were heteroscedastic in nature were transformed and the transformation revealed more stationary as shown in Table 2 (McAleer, 2014; McAleer & Hafner, 2014). The transformed data revealed that it was heteroscedastic in nature because the data showed volatility clustering, skewness and kurtosis. The heteroscedasticity tests of ARCH and White tests were implemented to observe the heteroscedasticity effect, and the *F*-Statistic and Obs*R-squared were significant which indicated that there were ARCH in the data. ARCH presence in the data was the reason for employing GARCH family models.

Table 1: Normality Tests for Real Gross Domestic Product (RGDP) Data

	Standard deviation	Skewness	Kurtosis	Jarque-Bera
RGDP	164450.	-0.1971	2.3434	6.2326 (0.04430)**

P-values () *** significant at 1%, ** significant at 5%, * significant at 10%

Table 2: Statistical Summary, Normality and ARCH Tests for RGDP Data

		RGDP
Standard deviation		7.9627
Skewness		-0.9703
Kurtosis		2.5632
Jarque-Bera		5.2256 (0.0133)**
ARCH Test	F-Statistic	16.5550 (0.0003)***
	Obs*R-squared	11.7889 (0.0006)***
	F-Statistic	1.45E+24 (0.0000)***
White Test	Obs*R-squared	37.0000 (0.0000)***

P-values () *** significant at 1%, ** significant at 5%, * significant at 10%

The above tests revealed that Nigeria quarterly RGDP is heteroscedastic in nature. Thus, ARCH, GARCH, TGARCH, EGARCH, PARCH and CGARCH were heteroscedasticity(ARCH series) models well thought-out to obtain the best model forecast. The heteroscedasticity models were employing Maximum Likelihood estimators assuming Gaussian distribution (N), Student *t* Distribution (ST) and Generalized Error Distribution (GED). Gaussian (normal) distribution explanation is when the mean of the variable is zero and autocovariances are zero. Student *t* distribution explanation is when the degrees of freedom also called an additional parameter changes its form from standard normal distribution is known as Student *t* distribution. Generalized error distribution specification is when the coefficients of mean equations are significant.

Table 3 shown that among the heteroscedasticity models employed, only EGARCH-GED has the least minimum information criteria [Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) and Hannan-Quinn Information Criteria (HQ)] and highest value of loglikelihood (LL) revealed the best fit and good distribution fitheteroscedasticity model respectively. The value of Durbin-Watson (DW) statistics showed that there was no first order serial correlation.

Tables 4 and 5 showed that the dynamic forecast evaluation of CGARCH-GED has minimum

Root Mean Square Error (RMSE)value among the heteroscedastic models, EGARCH-ST has minimum Mean Absolute Error (MAE) value and minimum Mean Absolute Percentage Error (MAPE) value. RMSE, MAE and MAPE were the forecast errors that regulated the forecast accuracy when compared with other models forecast. The best forecast accuracy was revealed through the model that has the minimum forecast errors. The average of forecast error revealed that EGARCH-ST has minimum forecast error which was the best forecast accuracy and it indicated the best model forecast.

Thei inequality coefficient showed the fitness of the forecast errorbut all the models showed low fitness. The bias proportion values for models revealed fairly good forecast, because there was little biasness. The forecastswere not following the variant in the actual RGDP series, these were the effect of large variance proportion (VAR). For a good forecast, the covariance proportion (COV) supposed to be large but the values were low.

The dynamic forecast employs the forecast value of lagged dependent variables while the static forecast employs the actual value of the explanatory variables in the production of forecast.

Table 3: Order of ARCH series of models Using RGDP Data

Estimation	AIC	BIC	HQ	LL	DW
ARCH	6.6253	6.8095	6.6967	-118.75	2.1738
GARCH-N	6.7075	6.9252	6.7843	-119.09	2.1529
GARCH-ST	6.7211	6.9826	6.8132	-118.34	2.1285
GARCH-GED	6.8302	7.0914	6.9223	-120.36	2.1562
TARCH-N	6.7410	7.0022	6.8331	-118.71	2.1481
TARCH-ST	6.1809	7.1157	6.9184	-119.00	2.1474
TARCH-GED	6.6802	6.9850	6.7877	-116.59	2.1946
EGARCH-N	5.5993	5.8605	6.6914	-97.587	2.0119
EGARCH-ST	5.6626	5.9673	5.7700	-97.757	2.0151
EGARCH-GED	5.4517	5.7564	5.5591	-93.856	2.0086
PARCH-N	6.8634	7.1682	6.9708	-119.97	2.1893
PARCH-ST	6.8923	7.2466	7.6151	-119.51	2.1445
PARCH-GED	6.8036	7.1519	6.9264	-117.80	2.1191
CGARCH-N	6.7335	7.0383	6.8469	-117.57	2.1575
CGARCH-ST	6.7209	7.0692	6.8437	-116.33	2.1533
CGARCH-GED	6.6410	6.9894	6.7638	-114.86	2.1561

Table 4: Dynamic forecast for Nigeria RGDP

Estimation	RMSE	MAE	MAPE	THEI	BIAS	VAR	COV
ARCH	7.9647	6.6541	90.955	0.9334	0.0007	0.9906	0.0083
GARCH-N	7.9982	6.1543	74.281	0.9334	0.0098	0.9656	0.0046
GARCH-ST	8.0753	5.8834	65.838	0.7985	0.0216	0.9696	0.0013
GARCH-GED	7.9764	6.2984	79.073	0.8633	0.0040	0.9890	0.0070
TARCH-N	8.0200	6.6523	70.997	0.8243	0.0154	0.9815	0.0031
TARCHST	8.0220	6.6468	70.429	0.8231	0.0160	0.9811	0.0030
TARCH-GED	7.9661	6.3693	81.496	0.7578	0.0021	0.9949	0.0030
EGARCH-N	8.4341	5.5952	59.658	0.7314	0.1086	0.8888	0.0012
EGARCH-ST	8.4277	5.5941	59.574	0.7321	0.1086	0.8901	0.0012
EGARCH-GED	8.5707	5.6484	62.124	0.7195	0.1376	0.9879	0.0066
PARCH-N	8.0023	6.1211	73.224	0.8344	0.0114	0.9781	0.0022
PARCH-ST	8.0371	5.9947	69.200	0.8149	0.1974	0.9629	0.0007
PARCH-GED	8.1055	5.8101	63.648	0.7883	0.0365	0.9892	0.0070
CGARCH-N	7.9753	6.3073	79.368	0.8648	0.0034	0.9892	0.0090
CGARCH-ST	7.9689	6.3818	81.837	0.7883	0.0018	0.9821	0.0090
CGARCH-GED	7.9637	6.4491	84.081	0.8909	0.0007	0.9896	0.0097

Table 5: Static forecast for Nigeria RGDP

Estimation	RMSE	MAE	MAPE	THEI	BIAS	VAR	COV
ARCH	7.9142	6.5403	86.111	0.8997	0.0007	0.8618	0.1375
GARCH-N	7.9575	6.0742	72.102	0.8202	0.0116	0.8523	0.1362
GARCH-ST	8.0550	5.8602	65.712	0.7899	0.0319	0.8951	0.0730
GARCH-GED	7.9272	6.1851	75.434	0.8348	0.0050	0.8248	0.1703
TARCH-N	7.9867	5.9970	69.769	0.8100	0.0176	0.8724	0.1100
TARCH-ST	7.9892	5.9914	69.588	0.8091	0.0182	0.8735	0.1084
TARCH-GED	7.9376	6.3039	78.776	0.8676	0.0023	0.9320	0.0658
EGARCH-N	8.4355	5.5328	56.680	0.7371	0.1023	0.8276	0.0701
EGARCH-ST	8.4297	5.5327	56.624	0.7379	0.1010	0.8280	0.0710
EGARCH-GED	8.5704	5.6105	58.341	0.7324	0.1145	0.7159	0.1695
PARCH-N	7.9940	6.1036	72.576	0.8325	0.0118	0.9695	0.0187
PARCH-ST	8.0094	5.9478	68.225	0.8037	0.0220	0.8873	0.0907
PARCH-GED	8.0912	5.7999	63.875	0.7823	0.0389	0.9076	0.0535
CGARCH-N	7.9260	6.1936	75.672	0.8366	0.0046	0.8270	0.1684
CGARCH-ST	7.9157	6.2537	77.566	0.8410	0.0023	0.8014	0.1962
CGARCH-GED	7.9099	6.3185	79.528	0.8553	0.0009	0.8023	0.1968

Conclusion

The graph of level data displayed tending and not stationary. The tests conducted on transformed data revealed the suitability of heteroscedasticity model. The results of the analysis revealed that the data has good distribution fit with highest loglikelihood and the best model fit with the least minimum information criteria in EGARCH-GED among the heteroscedasticity models (Almeida & Hotta, 2014).

The dynamic and static forecast evaluation results revealed that EGARCH-ST was the best

forecast accuracy and the best model forecast (Mutunga *et al.* 2015). The inequality coefficients showed low fitness, little biasness proportions, large variance proportions and low values of covariance proportions which showed fairly good forecast.

Hence, EGARCH-ST is the best model forecast for forecast accuracy using Nigeria RGDP quarterly data for better decision that can boost the economy of the society.

REFERENCES

- Almeida, D. D., & Hotta, L. K. (2014). The leverage effect and the asymmetry of the error distribution in GARCH-based models: the case of Brazilian market related series. *Pesquisa Operacional*, 34(2), 237-250. doi.org/10.1590/0101-7438.2014.034.02.0237
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of econometrics*, 31(3), 307-327. doi.org/10.1016/0304-4076(86)90063-1
- Chang, C. L., and McAleer, M. (2017). The correct regularity condition and interpretation of asymmetry in EGARCH. *Economics Letters*, 161, 52-55.
- Chang, C. L., and McAleer, M. (2019). The fiction of full BEKK pricing fossil fuels and carbon emissions. *Financial Research Letters*, 28, 11-19.
- Ding, Z., Engle, R. F. and Granger, C. W. J. (1993). A Long Memory Property of Stock Market Return and a New Model. *Journal of Empirical Finance*, 1, 83-106.
- Dynan, K., and Sheiner, L. (2018). "GDP as a measure of economic well-being." Work. pap, 43.
- Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica: Journal of the Econometric Society*, 987-1007. DOI: 10.2307/1912773
- Ewing, B. T. and Malik, F. (2013). Volatility transmission between gold and oil futures under structural breaks. *International Review of Economics & Finance*, 25, 113-121. doi.org/10.1016/j.iref.2012.06.008
- Feldstein, M. (2017). Underestimating the real growth of GDP, personal income, and productivity. *Journal of Economic Perspectives*, 31, 2, 145 - 164
- Harvey, A. C. (1993). *Time series models* 2nd edition. The MIT Press. Cambridge. Massachusetts. Harvester Wheatsheaf Publisher.
- Hentschel, L. (1995). All in the family nesting symmetric and asymmetric GARCH models. *Journal of Financial Economics*, 39, 1, 71-104. doi.org/10.1016/0304-405X(94)00821-H
- Jiang, Z. and Xia, W. (2018). Volatility Modelling with Leverage Effect under Laplace Errors. *Journal of Time Series Econometrics*, 10, 1.
- Martinet, G. G. & McAleer, M. (2016). On the Invertibility of EGARCH (p, q). *Econometric Reviews*, 1-26. doi.org/10.1080/07474938.2016.1167994
- McAleer, M. (2014). Asymmetry and Leverage in Conditional Volatility Models. *Econometrics*, 2, 3, 145-150. doi:10.3390/econometrics2030145
- McAleer, M. & Hafner, C. M. (2014). A oneline derivation of EGARCH. *Econometrics*, 2, 2, 92-97. doi:10.3390/econometrics2020092
- Mutunga, T. N., Islam, A. S. & Orawo, L. A. O. (2015). Implementation of the estimating functions approach in asset returns volatility forecasting using first order asymmetric GARCH models. *Open Journal of Statistics*, 5, 05, 455. DOI:10.4236/ojs.2015.55047
- Nelson, D. B. (1991). Conditional heteroscedasticity in asset returns: A new approach. *Econometrica: Journal of the Econometric Society*, 347-370. DOI: 10.2307/2938260
- Ngo Thai, H. (2019). Dynamics of Volatility Spillover Between Stock and Foreign Exchange Market: Empirical Evidence from Central and Eastern European Countries. *Economy and Finance*, 6, 3, 244-265 Doi: 10.33908/EF.2019.3.
- Rodríguez, M. J. & Ruiz, E. (2012). Revisiting several popular GARCH models with leverage effect: Differences and similarities. *Journal of Financial Econometrics*, 10, 4, 637-668. DOI: https://doi.org/10.1093/jjfinec/nbs003
- Thapa, B. S., and Gautam, C. M. (2016). Stock Return Volatility and Asymmetries in Stock Market of Nepal.
- Vivian, A. & Wohar, M. E. (2012). Commodity volatility breaks. *Journal of International Financial Markets, Institutions and Money*, 22, 2, 395-422. doi.org/10.1016/j.intfin.2011.12.003

Zakoïian, J. M. (1994). Threshold Heteroscedastic Models. *Journal of Economic Dynamics and Control*, 18, 931–955.