



## Operational Amplifier: A Simultaneous Linear Equation Solver

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### ABSTRACT

Operational Amplifier (Op-Amp) as it is popularly called is used to solve simultaneous linear equations. A pair of simultaneous linear equation with two unknowns has been considered for the design of the balanced adder-subtractor operated from a 6-Vd.c power source. A general purpose op-amp,  $\mu A$  741 of the 8 pin mini DIP type was chosen for the design. Its unique features and prime advantages was considered and hence its choice. The performance of the designed circuit has been evaluated for the input voltages, 20V and 5V respectively. The output obtained from the system,  $X_0 = 0.623V$  and  $Y_0 = 0.754V$  On the scale of 0.1V to 1 confirmed the solution of the pair of simultaneous equation.

**Keywords:** Op-Amp, linear equation, adder-subtractor

### INTRODUCTION

Operational amplifiers are widely used in electronic and measurement systems. They are among the active components of electronic circuits. Its applications in electronic circuits have received much attention due to their potential advantages in terms of high frequency performance, ease of design and miniaturization (Shinde, 2002). They can be configured to execute most mathematical operations such as integration, differentiation, simultaneous linear and quadratic equations among others.

A balanced adder-subtractor is a circuit that employ the use of operational amplifier to solve simultaneous linear equation and some other related operations. The circuit built here is a generalization of the differential input amplifier. Its operation is based on summation and subtraction of the voltages. In its best operating conditions, the inverting and non-inverting gains must be balanced, and hence the name (Donald, 2001).

A linear equation in two variables of the form  $ax + by = c$  has no unique solution. This is an equation of a straight line which would be satisfied with several points with coordinates (x, y) on the line (Clayton, 1987). However, for a pair of linear equation with two variables, a unique solution can be obtained so long as they are not of parallel line equations. Several methods have been devised in solving a pair of simultaneous linear equations among which includes graphical, elimination and substitution

methods, while some are easily solved by simple manipulation some are not that simple and easy

(Asaolu, 1995) worked on circuits for solving simultaneous linear equation, this circuits uses operational amplifier as an inverting amplifier to which balance was not achieved, more so, it was designed for a specific cases.

This work is designed to achieve a balanced adder – subtractor, an analog circuit that would help in solving pairs of simultaneous linear equations. A pair of linear equation:  $2x + 3y = 40$  and  $2x + y = 5$  is considered for this work. The circuit built for this would however be modified for a general purpose.

A generalized circuit employs an operational amplifier (741) as the active component as shown below figure 1.

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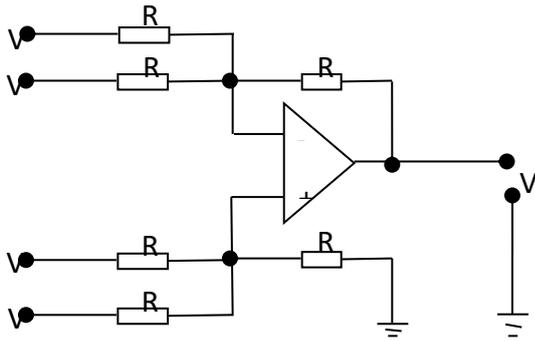


Figure 1: A generalized balanced

**DESIGN OF A BALANCED ADDER-SUBTRACTOR CIRCUIT**

In a balanced adder – subtractor, the loop gain in both directions is equalled (i.e. inverting and non-inverting).

Considering figure 1, therefore,

$$\frac{R_f}{R_1} + \frac{R_f}{R_2} + \dots + \frac{R_f}{R_m} = \frac{R'_f}{R'_1} + \frac{R'_f}{R'_2} + \dots + \frac{R'_f}{R'_n} \dots \dots \dots (1)$$

The output voltage  $V_o$  in both cases is given as

$$V_o = \left( V'_1 \frac{R'_f}{R'_1} + V'_2 \frac{R'_f}{R'_2} \right) - \left( V_1 \frac{R_f}{R_1} + V_2 \frac{R_f}{R_2} \right) \dots \dots \dots (2)$$

The pair of simultaneous linear equation

$$2x + 3y = 40 \dots \dots \dots (3)$$

$$2x + y = 5 \dots \dots \dots (4)$$

Can be expressed as

$$x = 20 - 1.5y \dots \dots \dots (5)$$

$$y = 5 - 2x \dots \dots \dots (6)$$

Equation (5) and (6) shows that, the mathematical operation in solving for X and Y is that of addition and multiplication by a constant coefficient which can be achieved using electrical signals.

In the computation of analog, the variables are represented continuously by voltages. In this case, a scale of 0.1V equals 1V is chosen for it will contain all possible solutions for our simultaneous linear equation. X = 15 would therefore equal 1.5V at X – output region.

With the chosen scale, equation (5) becomes

$$X = 20 \frac{R'_f}{R'_1} - \frac{R_f}{R_1} y \dots \dots \dots (7)$$

A positive number is fed in on the non inverting input with gain set at unity i.e.  $\frac{R'_f}{R'_1} = 1$ . Therefore,  $R'_f = R'_1$  and so, for ease, equals 100KΩ and  $R_1 = 68KΩ$ .

The values obtained forms the bases for the first stage of the amplifier as shown in figure 2. Another resistor  $R_x$  was introduced at the non-inverting input to achieve a gain of 5, i.e.  $\frac{R'_f}{R'_1} = 5$ . Thus, the gain for both inverting and non-inverting equal 1.5;

$$R_x = 200KΩ.$$

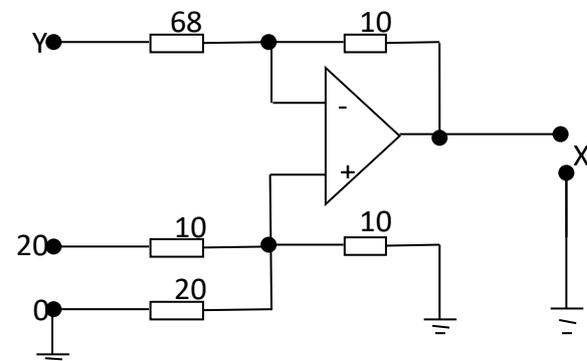


Figure 2: A circuit for X = 20 – 1.5y.

The second stage of the amplifier is set to solve for y. Since  $Y = 5 - 2x$ , it follows that

$$y = 5 \frac{R'_f}{R'_1} - \frac{R_f}{R_1} x \dots \dots \dots (8)$$

Where  $\frac{R_f}{R_1} = 2$  and  $\frac{R'_f}{R'_1} = 1$ .  $R_f = R'_f = 100KΩ$ , hence  $R_1 = 47KΩ$  and  $R'_1 = 100KΩ$ . These two values give an inverting gain of 2 which is not equal to the non inverting gain of 1 as set. To achieve the set target balance, a zero input of gain 1 is added to non inverting side i.e.  $\frac{R'_f}{R'_y} = 1$ , hence  $R'_y = 100KΩ$ . Thus, the second stage of the amplifier is obtained as shown in figure 3.

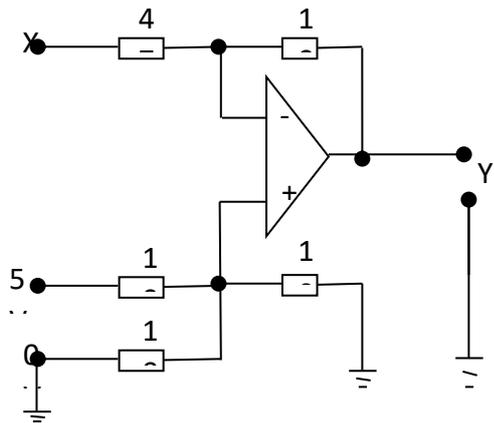


Figure 3: A circuit for  $Y = 5 - 2x$ .

For any pair of simultaneous linear equation, the solution of X and Y are interdependent. Hence the two stages of the amplifiers above (figure 2 and figure 3) are cascaded back – back such that the output of stage which solve for X is connected to serve as input to the X-input of the stage which solved for Y, while the output stage that solve for Y serves as the Y-input for solving x. This is represented in figure 4 shown below. The two stages of the electronic components were coupled on a single board as one circuit of the designed balanced summer – subtractor for solving a pair of simultaneous linear equations.

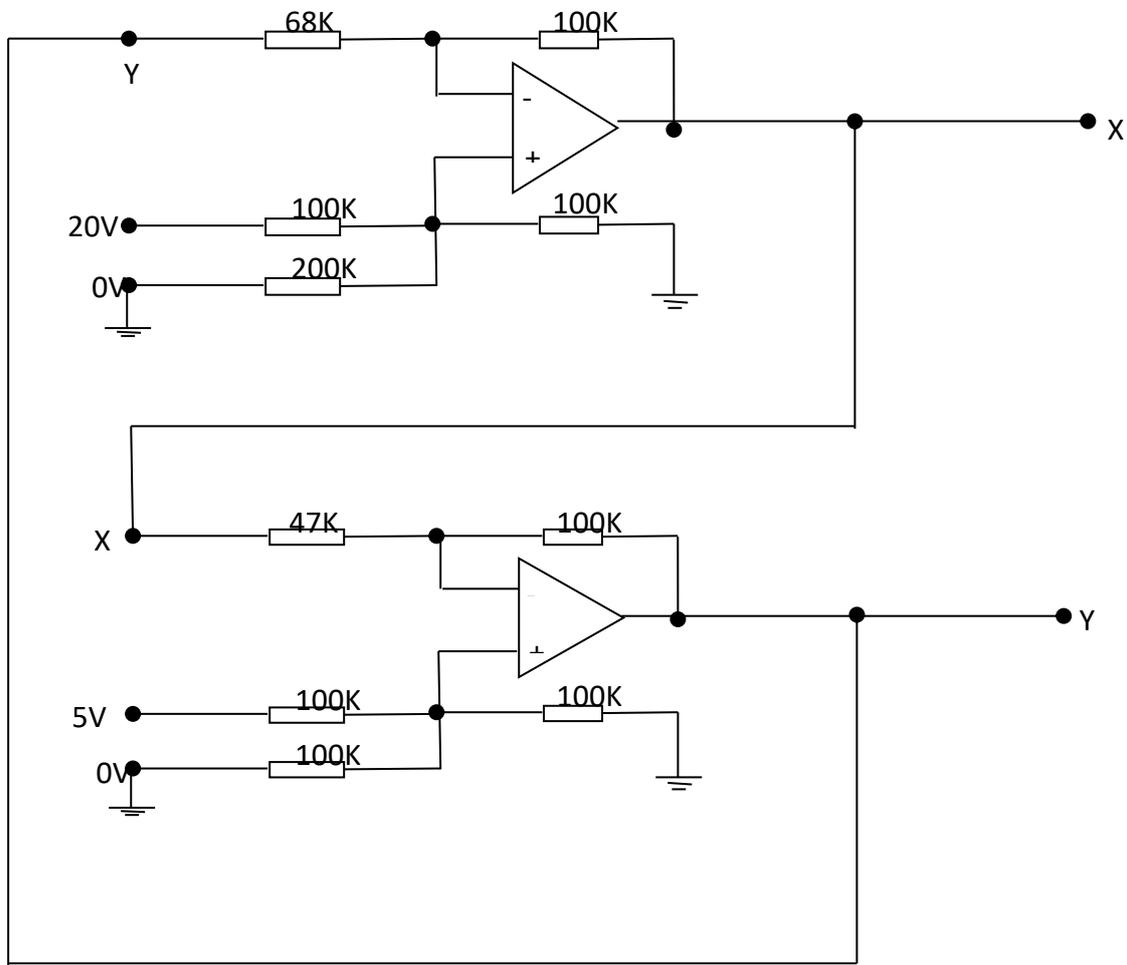


Figure 4: Circuit for solving simultaneous linear equation  $2x + 3y = 40$  and  $2x + y = 5$ .

## RESULTS AND DISCUSSION

Results obtained for the given pair of equation are stated in the table 1 below

$X_o$	$Y_o$	$X_a$	$Y_a$
0.623	0.754	6.25	7.50

$X_o$  and  $Y_o$  are the results obtained from the output of the circuit for X and Y respectively, while  $X_a$  and  $Y_a$  are the corresponding values obtained when solved mathematically. Noting the scale of 0.1V equals 1V chosen, it shows that the two solutions obtained are almost the same. The slight difference noticed might be as a result experimental random errors in the reading output signal on the oscilloscope.

### Conclusion

The circuit shows a favourable application in handling a pair of simultaneous linear equations. The circuit built helped to appreciate the versatility of operational amplifier and logic of op-amp as a summer and subtractor. The circuit used can be modified for a general application in solving a general set of any pair of simultaneous linear equation simply by changing  $R_{fs}$ ,  $R_x$  and  $R_y$  into variable precision resistors whose values can be varied to suit the desired cases.

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